ENGINEERING IN ENTERTAINMENT

Interactive Musical Displays, The Walking Piano

Solution:

We're given that the notes are played in middle **C**, which corresponds to C_4 to C_5 in pitch notation (openmusictheory.com/pitches.html). Next, we find the frequencies of the notes played from a table.
Here's the excerpt from one compiled in Michigan Tech's Physics course notes (pages.mtu.edu/~suits/notefreqs.html):

C^4	261.63
$C^{\#}_4/D^b_4$	277.18
D_4	293.66
$D^{\#}_4/E^b_4$	311.13
E_4	329.63
F_4	349.23
$F^{\#}_4/G^b_4$	369.99
G_4	392.00
$G^{\#}_4/A^b_4$	415.30
A^4	440.00
$A^{\#}_4/B^b_4$	466.16
B_4	493.88
C_5	523.25

We can fill in the blanks for f in the sinusoids using the table. We need A, G, and F#, which correspond to 440, 392, and 369.99, respectively. We weren't given the amplitude, so we'll take A to be 1 to correspond to "full volume." This means our sinusoids are:

$$\{\sin(2\pi(440t)),\sin(2\pi(392t)),\sin(2\pi(369.99t))\}$$

Now, to have the A and G played together, we can add the sinusoids like so:

$$y(t)=\sin(2\pi(440t))+A\sin(2\pi(392t))$$

However, the F# was played two seconds late. Accordingly, we can use the Heaviside function to "turn on" the function at the right time. Our base Heaviside function looks like this:

$$u(t)=\begin{cases} 1, t\geq 0 \\ 0, t<0 \end{cases}$$

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(continued)

But we need it to start at t=2 seconds. This change can be easily done with the bounds by changing $t \geq 0$ to $t \geq 2$ and $t < 0$ to $t < 2$. The change in timing corresponds to *shifting* the Heaviside step function to the *right* by 2 units.

$$u(t-2) = \begin{cases} 1, t \geq 2 \\ 0, t < 2 \end{cases}$$

To complete the effect, we multiply the last sinusoid by our shifted Heaviside function to get:

$$\sin(2\pi(369.99t))u(t-2)$$

Now the F# contribution is not added until t=2, where the step function "turns on." Appending the modified sinusoid to our $y(t)$ yields the desired result:

$$y(t) = \sin(2\pi(440t)) + \sin(2\pi(392t)) + \sin(2\pi(369.99t))u(t-2)$$

Or, in simplified form:

$$y(t) = \sin(880\pi t) + \sin(784\pi t) + \sin(740\pi t)u(t-2)$$