



Interactive Musical Displays, The Walking Piano

Sample Problem:

When creating an interactive musical display, a crucial feature is processing the signals sent from the components in the display and outputting the appropriate sound. But first, we need to know how to represent the signals mathematically. A convenient way of transforming sounds into functions is using sinusoids, so a sound $x(t)$ can be written as

$$x(t) = A \sin(2\pi ft + \phi)$$

where A is the amplitude of the sound (often between 0 and 1 to represent no volume and full volume respectively), f is the frequency (Hz), t is the time variable (seconds), and ϕ is the phase shift (i.e., how far the function is shifted horizontally from its normal position).

Suppose a child jumps on the piano to play the notes G and A at the same time (in the middle C scale). A friend comes in to join and jumps on the F# key two seconds later. Write a function $y(t)$ that represents the notes played in sinusoidal form (i.e., using $x(t)$'s) at full volume.

Hint:

The Heaviside step function $u(t)$ can be helpful for “turning functions on and off” as needed. The step function is defined as:

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$