Engineering for Extreme Weather

Controlling circuits using logic

Problem

We see examples of electronics controlling devices and signals every day. When you are stopped at a red light, the other direction has a green light. When you visit your local barista, they push a button to tell the system the number of shots of espresso to brew. Electronic combination locks guard homes, cars, businesses, safes, etc. Some devices have very sophisticated electronic control: electric vehicle safety systems rely on collecting data external to the vehicle and adjusting steering, acceleration and braking. Space travel requires careful allocation of air and water with electronic circuitry controlling rates at which air and water are recycled or generated.

We can generate Boolean equations to specify control signals. Boolean equations use variables that are digital signals, which are 1 or 0 – similar to a light switch. These are combined using operators including AND (·), OR (+), and NOT (a bar over the quantity to be inverted - $\overline{\ }$).

Not: invert the value	AND: 1 if all inputs are 1	OR: 1 if any of the inputs are 1
NOT 0 = 1	0 AND 0 = 0	0 OR 0 = 0
NOT 1 = 0	0 AND 1 = 1 AND 0 = 0	0 OR 1 = 1 OR 0 = 1
	1 AND 1 = 1	1 OR 1 = 1
NOT A = $\overline{A}$	A AND B = A·B	A OR B = A + B
	(note: sometimes, we leave out the dot showing AND and just write AB)	

More complex equations can be built:

FireAlarm = VisibleFlame OR (Smoke AND (Rooftop OR Window)) OR (PullAlarm) can be written as:

$F = V + (S \cdot (R + W)) + P$

In many cases, once an equation is designed, it may be able to be simplified. For example,

FireAlarm = (VisibleFlame AND Roof) OR (VisibleFlame AND Window) OR (VisibleFlame) OR (Smoke AND Roof) OR (Smoke and Window) OR (PullAlarm and VisibleFlame) OR (PullAlarm)

Or

$F = (V \cdot R) + (V \cdot W) + (V) + (S \cdot R) + (S \cdot W) + (P \cdot V) + P$ or
$F = VR + VW + V + SR + SW + PV + P$

This more complex equation can be simplified to the original equation.

There are rules which are very similar to those used in algebra which we can use to simplify Boolean equations.

Use the given laws to simplify the following Boolean equations:

Name	AND form	OR form
Identity law	$1A = A$	$0 + A = A$
Null law	$0A = 0$	$1 + A = 1$
Idempotent law	$AA = A$	$A + A = A$
Inverse law	$A\overline{A} = 0$	$A + \overline{A} = 1$
Commutative law	$AB = BA$	$A + B = B + A$
Associative law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive law	$A + BC = (A + B)(A + C)$	$A(B + C) = AB + AC$
Absorption law	$A(A + B) = A$	$A + AB = A$
De Morgan's law	$\overline{AB} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A}\,\overline{B}$

Question #1:

$Z = Y(V+Y) + W + VY$

Question #2:

$F = (V \cdot R) + (V \cdot W) + (V) + (S \cdot R) + (S \cdot W) + (P \cdot V) + P$