November Monthly Math Challenge
High School Level
Problem & Solution

Instructions: TEAMS coaches submit student answers to the question(s) below using the submission link on the TEAMS website. All submissions must be made during the month of November. Those submissions with correct answers will be entered into a drawing for a $25 Visa gift card, which will be sent to the student in care of the TEAMS coach.

Hydroelectricity

Turbines are a central mechanism for hydroelectric systems. Turbines spin as water flows through while flowing from upstream, turning a generator to produce electricity; very similar to the way wind power turbines function when air passes through them. Depending on the topology, hydropower stations can capture the water flowing and possibly control when it goes through the turbine. Run-of-the-river topologies of hydropower will simply channel water through the turbine without any storage of water, whereas other topologies will either have water stored someplace or will use a pumped storage system that raises water elevation during low-demand periods so that more energy can be stored and then drawn out at peak hours. Power ratings for hydroelectric turbines are based on the site as much as they depend on the water's mass flow rate through them and the difference in height between the start and end of the system, also known as the head.

P (Watts) = m * g * H_{net} * η

m = mass flow rate (kg/s)
g = 9.81 m/s^2
H_{net} = net "head" or height between the entrance and exit of the system = H_{gross}*0.9
η = system efficiency

Density of water = 1000 kg/m^3

Question 1

Imagine an experimental hydroelectric system that has a pumped storage reserve of water to draw from for peak demand hours. When it is drawn out, it goes through a separate set of turbines from the main dam area. The gross head has been modeled to be 12 feet when drawing from the reserve; assume that this value and the mass flow rate do not change as the reserve is depleted. The turbine system has an efficiency of 92%.

If the peak demand necessitates 10 MW to be generated in addition to the maximum generative capacity of all other providers in the area for six hours,

a) what is the mass flow rate in kg/s that needs to be achieved and
b) how many gallons of water does the reserve need to have ready as peak demand begins?

Solution:

The reserve needs to achieve a mass flow rate of
a) 336,576 kg/s and needs to contain about
b) 1.92 billion (or 1.92x10^9) gallons to satisfy peak demand.

a) Available to plug into the power equation are the variables P (10,000,000), g (9.81), H_{net} through the gross head (0.9*12*0.3048 [m/ft] = 3.292), and η (0.92). The value of mass flow rate is unknown, so that value may be solved:

   m = 10,000,000/(9.81*0.92*3.292) = 336,576 kg/s

b) We need to operate with this mass flow rate for each hour using water from the reserve. We must calculate the volume rate of water rather than mass rate for the final answer:

   336,576 (kg/s) / 1000 (kg/m^3) = 336.576 m^3/s

   1 liter is 0.001 cubic meters (nullifies the numerical change of the mass to volume conversion for water):

   336.576 (m^3/s) / 0.001 (m^3/l) = 336,576 l/s

   Per second rate to per hour rate and scale adjustment:

   336,576 (l/s) * 60 (s/min) * 60 (min/hour) = 1,211,674,141 l/hour = 1211.674 megaliters/hour

   For six hours:

   1211.674 (megaliters/hour) *6 hours = 7270.04 megaliters

   7270.04 megaliters = 1.92 billion gallons or 1.92x10^9 gallons