



January Monthly Math Challenge

High School Level

Problem & Solution

Instructions: TEAMS coaches submit student answers to the question(s) below using the submission link on the TEAMS website. All submissions must be made during the month of January. Those submissions with correct answers will be entered into a drawing for a \$25 Visa gift card, which will be sent to the student in care of the TEAMS coach.

Forces and Friction

Newton's Laws are basic tools physicists and engineers use to analyze the forces that an object is subjected to. There are three laws, which are:

1. Every object in a state of uniform motion will remain in that state of motion unless an external force acts on it.
2. Force equals mass times acceleration:

$$f(t) = m a(t)$$

3. For every action there is an equal and opposite reaction: $F_{12} = -F_{21}$

Newton's first law that objects like to remain in whatever state they are in. For example, in the absence of gravity or resistance from the environment, a particle moving through space will continue to move until a force is exerted to stop it. Newton's second law states that the force an object exerts is the mass (matter) of the object multiplied by the rate at which its velocity is changing. The force of an object is the effect of the mass of that object and the change of how it moves through space (direction is included). The third law implies that the ground we stand on pushes back with a force that is equal to your weight.

Three common forces acting on an object are:

- *Gravitational force* (F_g) – this force acts directly downward, towards the Earth's core, and is a product of the mass and the acceleration due to gravity (9.81 m/s^2). The gravitational force is usually known as the weight of an object.
- *Normal force* (F_N) – contact force that acts on an object perpendicular to a surface with which it is in contact.
- *Frictional force* (F_f) – acts against the direction of motion, generally decreasing motion or net force due to interactions between the object and a surface

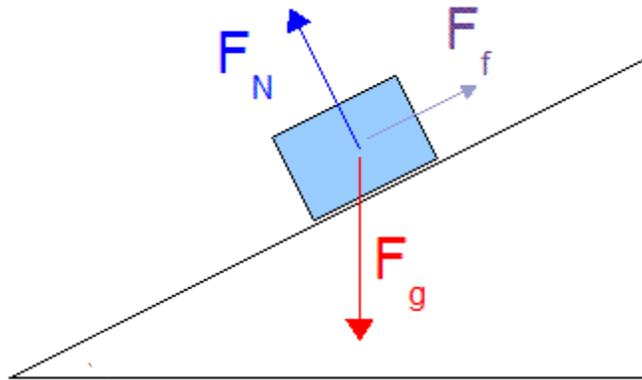


Figure 1: Free-body diagram of a block sitting on a ramp. The gravitational force (F_g) acts downward, the normal force (F_N) is perpendicular to the surface, and the frictional force (F_f) acts opposite the direction of potential motion.

Forces can be summed in terms of X and Y components to build a complete picture of the problem at hand.

Question

A 376 kg semi-truck with a trailer is accelerating at $2 \frac{m}{s^2}$. The truck is pulling the trailer up a 32.5-degree slope. The force of friction is 22 percent of the normal force exerted by the slope. What force is the truck exerting?

Solution: 1713 N

$$\text{Total force} = F_T = F_{truck} - F_f = \text{trailer mass} \cdot \text{acceleration} = 376(\text{kg}) \cdot 2(\text{m/s}^2) = 752 \text{ N}$$

Find the sum of forces in Y direction to find F_N :

$$F_{N_y} = F_N \cdot \cos(32.5) = F_g = m \cdot g$$

$$F_N = \frac{mg}{\cos(32.5)} = \frac{376 \cdot 9.8}{\cos(32.5)} = \frac{3684}{0.8434} = 4369 \text{ N}$$

$$F_f = 0.22 \cdot F_N = 0.22 \cdot 4369 = 961 \text{ N}$$

$$F_{truck} = F_T + F_f = 752 + 961 = 1,713 \text{ N}$$