December Monthly Math Challenge
High School Level
Problem & Solution

Instructions: TEAMS coaches submit student answers to the question(s) below using the submission link on the TEAMS website. All submissions must be made during the month of December. Those submissions with correct answers will be entered into a drawing for a $25 Visa gift card, which will be sent to the student in care of the TEAMS coach.

Natural Systems

When our conventional tools fail in design and analysis, we look to nature for inspiration. Although the approaches we use often do not give us the exact answer, our tools based on natural systems let us crack into enduring problems in mathematics, engineering, and computer science. For example, ant colonies have provided us with inspiration for solving difficult optimization problems. The ant colony optimization technique is a type of process that belongs to a metaheuristic to problem-solving techniques called simulated annealing. If you are more concerned with finding a "good enough" solution to a problem instead of the best because of time-constraints, computational limitations, or shear difficulty in solving the problem exactly, then simulated annealing is a suitable approach. This helps us avoid checking an enormous number of possible solutions.

Question 1

Suppose you were trying to design a transportation network that involves 100 towns. You want a path that passes through all the towns exactly once to avoid backtracking and returns to the starting point. How many unique solutions are possible?

Solution

Begin the solution by pretending to pick a town to start, then progressively build the path. If you begin with a randomly chosen town, there are 99 towns left to pick from. Pick another town, leaving 98. The next pick leaves 97. Continuing this process creates the following expression using the fundamental counting principle:

$$\#\text{Solutions} = (100)(99)(98)(97) \ldots (3)(2)(1) = 100!$$

However, we overcounted. The solution of moving one direction from town 1 to 100 is the same as moving from town 100 to 1. Therefore, we need to divide our count by 2.

$$\#\text{Solutions} = 100!/2 = 4.67e{+}157 = 4.67 \times 10^{157}$$